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Spontaneous Parity Violation in a Quantum Spin Chain

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Abstract. We report on a spontaneous breakdown of parity in the ground state of a spin Hamiltonian involving nearest-neighbor interactions. This occurs for a one-dimensional model where spins transform under the gauge field representation of QCD, the eight-dimensional adjoint representation of SU(3). The ground state spontaneously violates parity and is two-fold degenerate. In addition, the model possesses a non-vanishing topological string order parameter which we explicate analytically.

1. Introduction

Symmetries and conservation laws constitute an essential ingredient of physical theories [1]. An intriguing situation arises when a given microscopic model Hamiltonian is known to be symmetric under a certain type of transformation, but the expected regularity associated with this symmetry does not manifest itself in the physical properties. Beyond anomalous breaking, the only other mechanism known for this type of event is spontaneous symmetry breaking. There, the ground state of a system does not share the symmetries of the Hamiltonian. For continuous symmetries, a prominent example is the ferromagnetic phase in the Heisenberg model, where the ground state breaks the SO(3) rotational symmetry by singling out one certain magnetization axis.

In this article, we consider the discrete parity symmetry $\mathcal{P}$. Parity symmetry breaking (PSB) for the weak interaction has been a milestone in particle theory [2]. In the theory of magnetism, PSB has emerged in different contexts for higher dimensions, and is generally referred to the real space component of chiral symmetry breaking [3], which additionally includes time reversal symmetry breaking $\mathcal{T}$. However, it has remained elusive in most cases to define a model which spontaneously generates PSB. In the spin chain model which we discuss here, the algebraic structure of the adjoint representation of SU(3) at each site of the spin chain allows to accomplish this. In this contribution, we analyze the degenerate ground state manifold of this model and study some low-energy properties. Furthermore, we observe that the model possesses a topological string order which we calculate exactly.

2. The model and its properties

To construct a valence bond state with spins transforming under the eight-dimensional adjoint representation **8** of SU(3), we place a fundamental representation **3** and an anti-fundamental

representation $\bar{\mathbf{3}}$ of SU(3) on each lattice site. We then project the resulting tensor product onto the symmetric subspace, which yields the adjoint representation: $\mathcal{S}\{\mathbf{3} \otimes \bar{\mathbf{3}}\} = \mathbf{8}$. We generate an overall spin singlet state by coupling each representation $\mathbf{3}$ antisymmetrically with a representation $\bar{\mathbf{3}}$ on the neighboring site into a singlet bond: $\mathcal{A}\{\mathbf{3} \otimes \bar{\mathbf{3}}\} = \mathbf{1}$. This construction yields two linearly independent representation $\mathbf{8}$ states, Ψ^{L} and Ψ^{R}, which may be visualized as
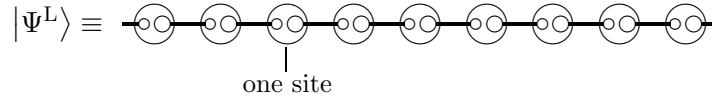
and its parity conjugate obtained by interchanging fundamental (small circles) and anti-fundamental representations (larger circles). The big circles indicate a lattice site and the horizontal lines between the sites are singlet bonds. As the construction is analogous to AKLT states [4], we may write the state vectors Ψ^{L} and Ψ^{R} as matrix product states. Taking (b,r,g) and (y,c,m) as bases for the representations $\mathbf{3}$ and $\bar{\mathbf{3}}$, respectively, we obtain the matrix [5]

$$
M_i = \begin{pmatrix}
\frac{2}{3}\left|\mathrm{by}\right\rangle_i - \frac{1}{3}\left|\mathrm{rc}\right\rangle_i - \frac{1}{3}\left|\mathrm{gm}\right\rangle_i & \left|\mathrm{ry}\right\rangle_i & \left|\mathrm{gy}\right\rangle_i \\
\left|\mathrm{bc}\right\rangle_i & -\frac{1}{3}\left|\mathrm{by}\right\rangle_i + \frac{2}{3}\left|\mathrm{rc}\right\rangle_i - \frac{1}{3}\left|\mathrm{gm}\right\rangle_i & \left|\mathrm{gc}\right\rangle_i \\
\left|\mathrm{bm}\right\rangle_i & \left|\mathrm{rm}\right\rangle_i & -\frac{1}{3}\left|\mathrm{by}\right\rangle_i - \frac{1}{3}\left|\mathrm{rc}\right\rangle_i + \frac{2}{3}\left|\mathrm{gm}\right\rangle_i
\end{pmatrix}.
$$

Assuming periodic boundary conditions (PBCs), the representation $\mathbf{8}$ states Ψ^{L} and Ψ^{R} are hence given by the trace of the matrix products

$$
\left|\Psi^{\mathrm{L}}\right\rangle = \mathrm{tr}\left(\prod_i M_i\right) \qquad \text{and} \qquad \left|\Psi^{\mathrm{R}}\right\rangle = \mathrm{tr}\left(\prod_i M_i^{\mathrm{T}}\right). \tag{1}
$$

These states transform into each other under space reflection or parity symmetry: $\mathcal{P}\Psi^{\mathrm{L}} = \Psi^{\mathrm{R}}$, where the discrete parity transformation is defined as $\mathcal{P} : \boldsymbol{S}_i \to \boldsymbol{S}_{N-i+1}$. Here $\boldsymbol{S}_i$ is an eight-component spin operator corresponding to the eight-dimensional representation of SU(3) and N denotes the number of lattice sites. Eigenstates of the parity operator with eigenvalues ± 1 are given by the even (symmetric) and odd (antisymmetric) superpositions $\Psi^{\pm} = \frac{1}{\sqrt{2}}\left(\Psi^{\mathrm{L}} \pm \Psi^{\mathrm{R}}\right)$.

A parent Hamiltonian which annihilates the states $\Psi^{\pm}$ is given by [6]

$$
\mathcal{H} = \sum_{i=1}^{N}\left(\boldsymbol{S}_i\boldsymbol{S}_{i+1} + \frac{2}{9}\left(\boldsymbol{S}_i\boldsymbol{S}_{i+1}\right)^2 + 1\right). \tag{2}
$$

The parity operator $\mathcal{P}$ commutes with the Hamiltonian, $[\mathcal{H}, \mathcal{P}] = 0$, while the ground states $\Psi^{\mathrm{L/R}}$ or $\Psi^{\pm}$ spontaneously violate this symmetry. The model (2) and its entropy were studied by Katsura *et al.* for open boundary conditions (OBC) and fixed dangling spins [7], while similar models with other symmetries were introduced in Refs. [8, 9, 10]. Starting from (2), it is natural to ask what happens for a representation $\mathbf{8}$ Heisenberg model with pure bilinear interactions $\boldsymbol{S}_i\boldsymbol{S}_{i+1}$. We find numerically that the ground state degeneracy is lifted when the prefactor of the biquadratic Heisenberg interaction is decreased from 2/9 to 0 (see Fig. 1). Preliminary finite size scaling data indicate that the splitting between the two lowest lying states of the Heisenberg model vanishes as we approach the thermodynamic limit $N \to \infty$, as required for a spontaneous symmetry violation. Above the two degenerate ground states, we expect a Haldane-type gap for the lowest lying excitations [5]. This conclusion is further supported by finite size scaling data not included here as well as the exponential decay of the static spin-spin correlations evaluated in the following section.
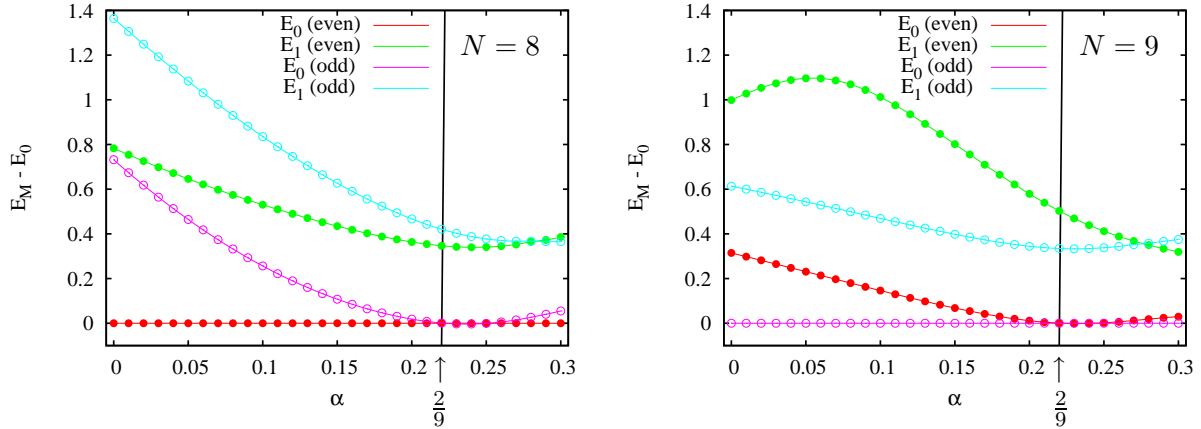
Figure 1. (Color online) Spectral flow of the two lowest energies of the two parity sectors (even/odd) under variation of the prefactor α of the biquadratic Heisenberg term in $\mathcal{H}(\alpha) = \sum_i \boldsymbol{S}_i \boldsymbol{S}_{i+1} + \alpha(\boldsymbol{S}_i \boldsymbol{S}_{i+1})^2$. PBCs have been imposed for both even ($N = 8$, left plot) and odd ($N = 9$, right plot) number of sites. The spectrum of the Heisenberg model differs considerably for even and odd number of sites. In the region around the exact model (2), $\alpha = 2/9$, the spectra become similar. As we move from (2) to the Heisenberg point, the ground-state degeneracy is lifted. For an even (odd) number of sites Ψ^+ (Ψ^-) becomes the ground state for $0 \leq \alpha < 2/9$.

3. String order parameter

In analogy to Refs. [10, 11, 12], we define the string operators

$$O_{1j}^{ab} = -J_1^a \exp\left[\mathrm{i}\,\pi \sum_{k=2}^{j-1} \left(J_k^3 + \frac{2}{\sqrt{3}} J_k^8 \right) \right] J_j^b, \quad a, b \in \{3, 8\}. \tag{3}$$

With the matrix product representations (1), the string order can be calculated using the method introduced by Klümper *et al.* [13] for the q-deformed model. The analysis proceeds in several steps. First, we calculate the norm of the matrix product state Ψ^{L}. We introduce the complex conjugated matrix $\tilde{M}$ according to $\tilde{M}_{\sigma\sigma'} = M_{\sigma\sigma'}^*$, *i.e.*, by taking the complex conjugate of each matrix element of M_i without transposing the matrix M_i. We then define the 9×9 transfer matrix R at any lattice site as

$$R_{\alpha\beta} = R_{(\sigma\tau),(\sigma'\tau')} = \tilde{M}_{\sigma\sigma'} M_{\tau\tau'}, \tag{4}$$

where we order the indices as $\alpha, \beta = 1, \ldots, 9 \leftrightarrow (11), (12), \ldots, (33)$. Finally, the norm is given by

$$\langle \Psi^{\mathrm{L}} | \Psi^{\mathrm{L}} \rangle = \mathrm{tr}\left(R^N \right) = \frac{1}{3^N}\left(8^N + 7(-1)^N \right) \to \left(\frac{8}{3} \right)^N, \tag{5}$$

where have evaluated the trace by diagonalization of R. Second, we introduce the transfer-matrix representation of the spin operators $J^{3,8}$ by

$$\hat{J}_{\alpha\beta}^{3,8} = \hat{J}_{(\sigma\tau),(\sigma'\tau')}^{3,8} = \tilde{M}_{\sigma\sigma'} J^{3,8} M_{\tau\tau'}. \tag{6}$$

Third, we introduce the operator $A = \tilde{M}\, e^{\mathrm{i}\,\pi(J^3 + 2J^8/\sqrt{3})}\, M$ such that

$$\langle O_{1j}^{33} \rangle_{\mathrm{L}} \equiv \frac{\langle \Psi^{\mathrm{L}} | O_{1j}^{33} | \Psi^{\mathrm{L}} \rangle}{\langle \Psi^{\mathrm{L}} | \Psi^{\mathrm{L}} \rangle} = -\frac{\mathrm{tr}\left(\hat{J}^3 A^{j-2} \hat{J}^3 R^{N-j} \right)}{\langle \Psi^{\mathrm{L}} | \Psi^{\mathrm{L}} \rangle} \xrightarrow{N \to \infty} \frac{9}{64}\left(1 + (-1)^j \frac{32}{8^j} \right) \xrightarrow{j \to \infty} \frac{9}{64}. \tag{7}$$

In the same way we obtain in the limit $N \to \infty$

$$\langle O_{1j}^{88} \rangle_{\mathrm{L}} = \frac{3}{64} \left(1 + (-1)^j \frac{224}{8^j} \right), \quad \langle O_{1j}^{38} \rangle_{\mathrm{L}} = \langle O_{1j}^{83} \rangle_{\mathrm{L}}^* = -\mathrm{i}\frac{3\sqrt{3}}{64} \left(1 - 8\frac{(-1)^j}{8^j} \right) \qquad (8)$$

as well as $\langle O_{1j}^{ab} \rangle_{\mathrm{R}} = \langle O_{1j}^{ba} \rangle_{\mathrm{L}}$. Note that the expectation values of the string order parameters remain finite in the limit $j \to \infty$. In analogy to the original AKLT model [14], we attribute this string order as well as the 18-fold degeneracy of the ground state of a chain with open boundary conditions to the violation of a discrete symmetry. Obviously, by choosing $A = R$, one obtains the static correlation functions $\langle J_1^a J_j^b \rangle = -\delta_{ab} (-1)^j \frac{27}{2} 8^{-j}$, *i.e.*, we find exponentially decaying correlations with correlation length $\xi = 1/\ln 8$.

4. Generalization to SU(n)
The model (2) is a special case of a model of SU(n) spins transforming under the $n^2 - 1$ dimensional adjoint representation $\mathcal{S}\{\boldsymbol{n} \otimes \bar{\boldsymbol{n}}\}$ with Hamiltonian

$$\mathcal{H}_{\mathrm{SU}(n)} = \sum_i \left(\boldsymbol{S}_i \boldsymbol{S}_{i+1} + \frac{2}{3n} \left(\boldsymbol{S}_i \boldsymbol{S}_{i+1} \right)^2 + \frac{n}{3} \right). \qquad (9)$$

The ground state is again two-fold degenerate and exhibits exponentially decaying correlations with $\xi = 1/\ln(n^2 - 1)$. Note that $n^2 - 1$ is just the dimension of the adjoint representation. We conjecture that the corresponding Heisenberg model has a two-fold degenerate ground state due to the broken parity symmetry as well.

5. Conclusion
In this contribution, we have established an example of spontaneous parity violation in a quantum spin chain. We further identified a non-local string order parameter for the model considered.

Acknowledgments

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